## Two Decades Dynamics of Belt Conveyor Systems

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#### **Summary**

The quest for a useful design tool that incorporated the effect of dynamics of conveyor belts on the design of a conveyor system started halfway the 1950's. It was however not until halfway the 1980's that the first useful design tool became available. In the early days of using dynamics of belt conveyor systems the attention was focussed on an analysis of both the starts and the stops of long overland, high tonnage/lift/speed conveyors. With the significant improvement of drive technology over the last twenty years however, it is now possible to start and (operationally) stop a belt conveyor in a very smooth manner. For an analysis of these non-stationary conditions the application of belt conveyor dynamics is not any longer required provided that sufficiently long starting and stopping times are used. The attention therefore shifted to the analysis of emergency stops and the determination of "what if" scenarios. It is an illusion to assume that a theoretical analysis gives all the answers. A practical verification of the results is of utmost importance to ensure that the assumptions made in the theoretical analysis were right and the advice given to the client correct.

This paper gives an overview of the work done on the mathematical description of dynamics of belt conveyor systems till date and briefly discusses the most important variables that effect belt conveyor dynamics. It will further give some practical recommendations and examples of the application of the belt conveyor dynamics in the design process of conveyor systems, the practical verification of the results and the lessons learned. Finally it will highlight the latest developments in the field and provide answers to frequently asked questions.

#### 1 Introduction

Due to the development of rubber technology, conveyor belts improved significantly after the Second World War and the application of belt conveyor systems for the transportation of bulk materials became widespread. Besides the application for in-plant transportation of bulk materials, the improved belt types enabled application in long overland systems as well. Therefore the capacity as well as the length of belt conveyor systems increased significantly.

To calculate the total power required for driving a belt conveyor system, design standards like DIN 22101 were, and still are, used. In these standards the belt is assumed to be an inextensible body. This implies that the forces exerted on the belt during starting and stopping can be derived from Newtonian rigid body dynamics, which yields the belt stress. With this belt stress the maximum extension of the belt can be calculated. This way of determining the elastic response of the belt is called the quasi-static (design) approach. For low capacity and small belt-conveyor systems this lead to an acceptable design and acceptable operational behaviour of the belt. However, up scaling of belt conveyor systems to high capacity or long distance systems introduced operational problems including:

- excessive large displacement of the weight of the gravity take-up device
- premature collapse of the belt, mostly due to the failure of the splices
- destruction of the pulleys and major damage of the idlers
- lifting of the belt off the idlers resulting in spillage of bulk material
- damage and malfunctioning of (hydrokinetic) drive systems

These problems, which did not occur in low capacity or short belt conveyor systems, triggered the quest for design tools that incorporated the effect of dynamics of conveyor belts on the design of a conveyor system.



#### 2 Modeling belt conveyor dynamics – a historic overview

Looking at the development of design tools that incorporate the effect of dynamics of conveyor belts, three periods can be distinguished. The first and perhaps most important period was from 1955 – 1975, the second period was from 1975 – 1995, and the last period from 1995 – today. The sections below give a brief overview of each period. For a more extensive overview see [1].

#### 2.1 The first period: 1955 – 1975

To detect the cause of the operational problems mentioned in the introduction, the behaviour of the belt and drive system during non-stationary operation were experimentally investigated. Relevant studies include [2]-[5]. The first objective of the experiments was to determine the development of axial (longitudinal) stress waves in the belt and their effect on the belt tension and the drive force. Also the delay effect, caused by the finite propagation speed of the stress waves in the belt, which appears during non-stationary operation of the belt, was of interest.

Oehmen [6] experimentally studied the start-up of a belt conveyor system in detail, accounting for the characteristics of the drive system, the tensioning system and the belt. The results of Oehmen's study were supplemented by Vierling [7] and confirmed by experiments done by others [8]-[10]. With the knowledge gained from these experiments the starting sequence of multiple-engine drive systems and the design of automatic tensioning systems could be improved, as has been described by a number of authors [11]-[15].

Field tests during starting and stopping of a belt conveyor are not always possible and the number of start-up and stop variations that can be tested is limited. Therefore the need for a mathematical model, which could be used to obtain detailed insight in the dynamic behaviour of a belt during non-stationary operation, increased. Because of the complexity of the equations of motion that describe the behaviour of a conveyor belt during non-stationary operation, the first mathematical models that were implemented for this purpose were the electrical analogue models, see for example [16]-[21]. Parallel to the development of these electrical analogue models, also analytical solutions of the equations of motion were developed. The first researchers that did an attempt to develop an analytical solution were Havelka [22] and Sobolski [23]. Their simplified model however was not very practical and other researchers tried to develop more functional models [24]-[29].

The first period was concluded by Funke [30] who developed the first really useful mathematical model. Funke discretised the belt into two homogenous and continuous elements that represented the carrying and the return strand of the belt. The global elastic response of the total belt was made up by the elastic response of the two elements. The motion of these elements was coupled through the motion of the pulleys. His model included time dependent motion resistances and he considered the visco-elastic character of the belt material. With the results obtained from calculations with Funke's model, the insight in the behaviour of the belt during non-stationary operation increased considerably. This enabled the improvement of the design of high capacity or long distance belt conveyor systems.

It was then recognised that discretisation of the belt in more than two belt parts would increase the accuracy of the calculations. Instead of using one or two elastic elements with homogenous mass distribution, the belt should be modelled by a number of finite elements to account for the variations of the resistances, masses and forces exerted on the belt. However, the application of this kind of models required advanced computational equipment that was not available at the end of the sixties [31]. Besides Funke, also Rao [32] and Harrison [33] used computers to study the development of transient stresses during starting and stopping of the belt.

### 2.2 The second period: 1975 - 1995

Funke's model [30] basically was a very rudimental finite element model with two elements. Nordell and Ciozda [34] developed the first finite element model of a belt conveyor system



that was a significant step forward compared to the model of Funke. Their model included the time dependent drive force, motion resistances and visco-elastic behaviour of the belt material. Simulation of the dynamics of belt conveyors is one thing, visualisation of the results is another. Morrison [35] illustrated the power of applying computer graphics to visualise the simulation results. Verification of the simulation results has shown that finite element models of belt conveyors are quite successful in predicting the elastic response of the belt during starting and stopping, see for example [34], [36]-[40]. Even today, these finite element models are still being developed to improve the description of for example the motion resistances [41].

#### 2.3 The third period: 1995 - today

All the models developed in the period 1975-1995 were made to study the dynamic behaviour of a conveyor belt in the axial or longitudinal direction. The models therefore were one-dimensional. An important effect that follows from the second (vertical or transverse) dimension and that has to be accounted for is the effect of the belt sag. Although it is possible to include the effect of the belt sag on the propagation of axial stress waves in a one dimensional model (see [1]) all finite element models as mentioned in section 2.2 determine only the longitudinal elastic response of the belt. They therefore fall short in the accurate determination of:

- the motion of the belt over the idlers and the pulleys
- the dynamic drive phenomena
- the bending resistance of the belt
- the development of stress waves with steep gradient of stress rate
- the interaction between the belt sag and the propagation of longitudinal stress waves
- the interaction between the idler and the belt
- the influence of the belt speed on the stability of motion of the belt
- the dynamic stresses in the belt during passage of the belt over a (driven) pulley
- the influence of parametric resonance of the belt due to the interaction between vibrations
  of the take up mass or eccentricities of the idlers and the transverse displacements of the
  belt
- the development of standing transverse waves
- the influence of the damping caused by bulk material and by the deformation of the crosssectional area of the belt and bulk material during passage of an idler
- the lifting of the belt off the idlers in convex and concave curves

The transverse elastic response of the belt is also often the cause of breakdowns in long belt-conveyor systems and should therefore be taken into account. The transverse response of a belt can be determined with special (isolated) models as proposed in [42]-[43]. It is however more convenient to extend finite element models of belt conveyors with special two-dimensional elements as used in multibody dynamics that take this response into account. Lodewijks [1] & [44] developed the first finite element model of a belt conveyor using multibody dynamics. Other researchers later followed a similar approach using multibody dynamics software packages [45]. Today, finite element models of belt conveyor systems incorporating components of multibody dynamics can be considered as state-of-the-art.

#### 2.4 The future

Since today two-dimensional models are state-of-the-art the question is whether or not the future will bring full three-dimensional finite element models of belt conveyor systems. Adding the third dimension will give the opportunity to accurately study the dynamic behaviour of conveyor belts in horizontal curves. This is an important issue since the lateral movement of a belt over the idlers determines whether or not the selection of a certain curve radius or idler arrangement is admissible or not. Other important issues are belt tracking and the occurrence of bending belt stresses in the curves. These aspects could be studied using a full three dimensional model.



One drawback of using a full three dimensional model is that the size of the model, compared to a two-dimensional model, increases with a factor 10 to 20. This implies that even more computational power is required than is necessary for handling a two dimensional model. For comparison: a two-dimensional model has 1000 to 2000 times the number of degrees of freedom (variables) of a one-dimensional model, a three dimensional model will therefore have 10,000 to 40,000 times the degrees of freedom of a one-dimensional model. With the current progress in computational processing speed however, this will eventually not be the bottleneck. A more important bottleneck will be the complexity of the model. A simple one-dimensional finite element model is relatively easy to make, to use, and the results are relatively easy to understand. A three-dimensional model on the other hand is much more complicated and certainly (today and in the near future) not fit as a day-to-day engineering tool. Also, with increasing accuracy of the finite element model of the belt itself, the accuracy of models of other conveyor components like the motor, fluid coupling etc. becomes more important as well. It therefore remains to be seen whether or not three-dimensional finite element model will be applied to model belt conveyor systems.

An important development that will affect the future of belt conveyor systems is the application of dynamic simulation techniques in their control systems. Today, control systems of belt conveyors are rigid: they can control a start or stop procedure according to fixed procedures. Even in case of the application of a proportional brake system the braking procedure is rigid. although the brake setting can be changed. Current control procedures do not account for the circumstances under which a conveyor stopped, in case of a start, or started, in case of a stop. Today, for example most starts are either velocity or torque controlled. No difference is made between the start of a fully loaded belt or an empty belt, where as far as power and belt tension is concerned, there is an important difference. The same holds for example for the conditions under which a conveyor stopped. As far as the control procedures are concerned, no difference is made between a start after an emergency stop and a start after an operational stop. As far as belt tension distribution and sensitivity to the starting procedure is concerned there is an important difference. This implies that engineers are faced with the fact that they have to design control procedures that work more or less under all operational circumstances. These procedures however are far from optimal, they put the conveyor system in most (all normal) operational conditions under a relatively high strain just to make sure that they also work in the worst case conditions.

In the future, large belt conveyor systems will have intelligent control systems that for example keep track of the load on the belt and the operational status of the system. Both the status and load conditions can be recognised and the optimum operational procedures can be determined using the results of dynamic simulations that are performed on the spot. Feedback from the conveyor system using sensors in for example the belt, the pulley lagging, the tensioning system and the drive system ensure that the input parameters for the dynamic calculations are kept up to date. The result will be that on one hand safety factors of the conveyor's components can be (further) decreased and that on the other hand the overall system safety and reliability will increase. The research group of the author is already working on these smart control systems.

### 3 Important design parameters

For the description of the dynamic behaviour of conveyor belts it is important to know at what speed stress (or tension) waves travel through a belt. Two wave speeds can be distinguished:  $c_1$  and  $c_2$ . Wave speed  $c_1$  is the speed at which axial stress waves travel through a belt. Wave speed  $c_2$  is the axial propagation speed of transverse waves that determines the frequency of transverse vibrations of a belt supported by two idlers (belt flap frequency). Their definition is as follows:



$$c_{1} = \sqrt{\frac{E_{b}}{\rho_{b}}}$$

$$c_{2} = \sqrt{\frac{\sigma}{\rho_{b}}} = \sqrt{\frac{T}{\rho_{b}A_{b}}}$$
(1)

In the above given equations is  $E_b$  the Young's modulus of the belt,  $\rho_D$  the density of the belt,  $\sigma$  the belt stress,  $A_b$  the cross sectional area of the belt and T the belt tension. Typically the wave propagation speed  $c_1$  ranges from about 1,000 m/s for fabric belts to about 2,500 m/s for steelcord belts. Wave speed  $c_2$  typically varies from about 25 till 100 m/s. The wave speeds however, depend not only on the density of the belt material but on the weight of the bulk material on the belt and the load of the (reduced) masses of the supporting idlers as well.

If the two wave speeds are rewritten to:

$$c_1 = \sqrt{\frac{E_b}{\rho_b}} = \sqrt{\frac{E_b A_b}{m'_g}}$$

$$c_2 = \sqrt{\frac{T}{\rho_b A_b}} = \sqrt{\frac{T}{m'_g}}$$
(2)

where m'<sub>g</sub> is the belt weight per unit of length, then the effective wave speeds for an unloaded belt can be written as follows:

$$c_{1,\text{effU}} = \sqrt{\frac{E_b A_b}{m_g' + m_r'}} = \sqrt{\frac{m_g'}{m_g' + m_r'}} c_1 = C_U c_1$$

$$c_{2,\text{effU}} = c_2$$
(3)

where m'<sub>r</sub> is the reduced mass of the idlers per unit of length, and for a loaded, supported belt:

$$\begin{split} c_{1,\text{effL}} &= \sqrt{\frac{E_b A_b}{m_g' + m_1' + m_r'}} = \sqrt{\frac{m_g'}{m_g' + m_1' + m_r'}} \ c_1 = C_{L1} c_1 \\ c_{2,\text{effL}} &= \sqrt{\frac{T}{m_g' + m_1'}} = \sqrt{\frac{m_g'}{m_g' + m_1'}} \ c_2 = C_{L2} c_2 \end{split} \tag{4}$$

where  $m'_1$  is the weight of bulk material on the belt. In practice the equations (4) represent the lower limit of the speed calculation, and the equations (3) represent the upper speed limit. When the belt is not fully loaded and the actual belt capacity is  $C_{act}$  where the design capacity is  $C_{des}$ , then the effective wave speeds are:

$$c_{i,\text{eff}} = c_{i,\text{effU}} - \alpha \frac{C_{\text{act}}}{C_{\text{des}}} (c_{i,\text{effU}} - c_{i,\text{effL}}), i = 1,2 \text{ and } C_{\text{act}} \le C_{\text{des}}$$
 (5)



where  $\alpha$  is a load coupling factor and

$$C_{\text{des}} = A_{\text{bulk,des}} \rho_{\text{bulk}} V_b = m'_{\text{l,des}} V_b \text{ [kg/s]}$$
 (6a)

$$C_{act} = A_{bulk.act} \rho_{bulk} V_b = m'_{l.act} V_b [kg/s]$$
(6b)

where  $A_{\text{bulk}}$  is the cross sectional area of the bulk material on the belt,  $\rho_{\text{bulk}}$  the density of the bulk material and  $V_{\text{b}}$  the belt speed. Values of  $\alpha$  have been determined from field measurements for a number of belts carrying a variety of materials [42]. For fine, wet materials in contact with the belt, such as mineral sands, crushed bauxite and run-of-mine coal, the factor  $\alpha$  is typically in the range of  $0.8 < \alpha < 1$ . The lowest coupling factor determined is  $\alpha = 0.3$  for large lumps of dry rock ore (over 200 mm diameter) with a thin layer of dusty fine material on the belt. It is very important to accurately determine both wave speeds using the above given equations since their accuracy determined the accuracy of the dynamic calculations, as is clearly shown in [47].

#### 4 Modeling belt conveyor dynamics – an example

To illustrate the procedure of setting up a finite element model of a belt conveyor, lets consider the belt conveyor geometry shown in Figure 1. This belt conveyor consists of a belt, a drive pulley, a tail pulley, a vertical gravity take-up, a number of idlers and a plate support.

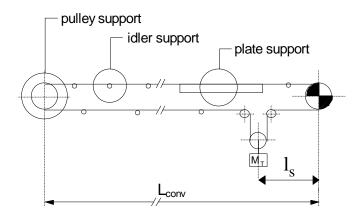
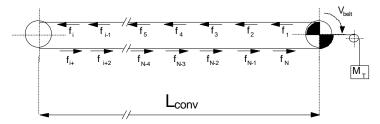


Figure 1: Typical belt-conveyor geometry indicating three support sections.

For a long belt conveyor system the length of the belt part between the drive pulley and the take-up pulley,  $I_{S}$ , is normally negligible compared to the length of the total belt,  $2L_{conv}$ . Therefore the simplification made by assuming that the take-up pulley and the drive pulley are located at the same position is justified. These two pulleys can mathematically be combined to one pulley as long as the mass inertia's of the pulleys of the take-up system are accounted for, see Figure 2.



**Figure 2:** Combination of the take-up pulley and the drive pulley. The distribution of the motion resistance forces is shown.

If the main interest is in the longitudinal elastic response of the belt then motion resistance forces can represent the effect of the belt support. These forces should account for the



motion resistance forces that normally act on the belt such as the resistance of the idlers to rotation on their bearings, the indentation rolling resistance etc. These forces vary from place to place depending on the exact local conditions and geometry of the belt conveyor and are therefore distributed along the length of the belt (Figure 2).

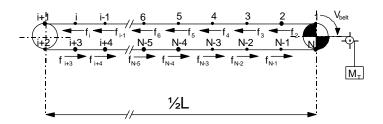


Figure 3: Belt divided into finite elements.

To be able to accurately determine the influence of these distributed forces on the motion of the belt, the belt is divided into a number of finite elements and the forces that act on a specific part of the belt are allocated to the corresponding element (Figure 3).

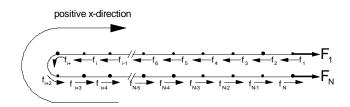
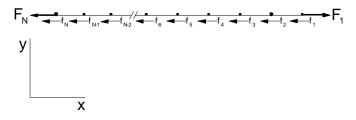


Figure 4: Replacing the interaction of the belt and its supports by forces.

If the interest is in the longitudinal elastic response of the belt only then the belt is not discretised on those places where it is supported by a pulley which does enforce its motion (no slip possible), for example the drive pulley. The belt is only discretised on those places where it is supported by a pulley that does not enforce its motion (slip possible) for example the non-driven tail pulley. This is shown in Figure 4.



**Figure 5:** Belt laid-out in x-direction.

The last step in building the model is to replace the belt's drive and tensioning system by two forces (the forces  $F_1$  and  $F_N$  in Figure 4) that represent the drive characteristic and the tension forces. These forces also account for the coupling between the two nodal points 1 and N. Forces allocated to the nodal points around that pulley account for the resistances in the bearings of the pulley and its mass inertia, see  $f_{i+1}$  and  $f_{i+2}$  in Figure 4.



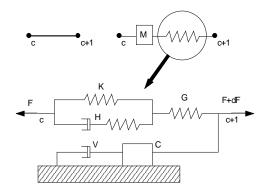


Figure 6: Five element composite model.

The maximum belt speed is about 10 m/s whereas the propagation speed of the longitudinal stress waves, as mentioned in section 3, varies from about 1,000 m/s for fabric belts to about 2,500 m/s for steelcord belts. Hence the influence of the belt speed on the axial elastic response of the belt is negligible. This implies that all the elements and forces that account for the interaction between the belt and its supporting structure remain in their position relative to the supporting structure. The value of these forces, however, varies from time to time. This depends on the temporary local belt speed and belt load that results from the distributed bulk material load that travels through the fixed element grid with the belt speed. In Figure 4 the orientation of the elements is given which results in the configuration of Figure 5 when the belt is laid in the x-direction.

The exact interpretation of the finite elements depends on which resistances and influences of the interaction between the belt and its supporting structure are taken into account and the mathematical description of the constitutive behaviour of the belt material. Depending on this interpretation, the elements can be represented by a system of masses, springs and dashpots as is shown in Figure 6 [34] where such a system is given for one finite element with nodal points c and c+1. The spring K and dashpot H represent the visco-elastic behaviour of the belt's tensile member. The spring G represents the belt's variable longitudinal geometric stiffness produced by the vertical acting forces on the belt's cross section between two idlers. The sliding friction element C represents the transitional static to dynamic friction and the dashpot V represent the belts velocity dependent resistances. The belt length represented by one element lies in the range of 10 till 250 m depending on the total belt length and the desired accuracy. A mathematical of the above illustrated model can be found in [1] or [44].

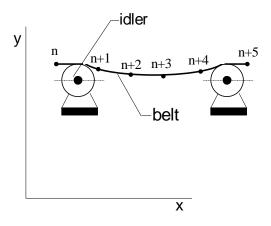


Figure 7: Belt supported by idlers.



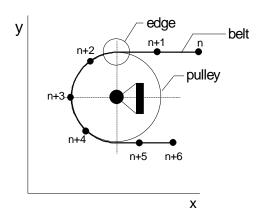


Figure 8: Belt supported by a pulley.

If, besides the axial elastic response, also the transverse elastic response is of interest, then the belt has to be discretised at such a scale that the motion of the belt over the supports can be determined. See for example the Figures 7 and 8 showing a belt section supported by idlers and a pulley. This implies that the belt length represented by one element in this case varies from 0.1 till 0.25 m, instead of 10 till 250 m for a one-dimensional model. Apart from the difference in belt length, the most important difference between models used to determine only the axial elastic response and models used to determine both axial and transverse response is that the position of the elements relative to the conveyor is not fixed for the latter. This is caused by the relative low axial propagation speed of transverse belt vibration, which is about 25 till 100 m/s. Compared to this speed the belt speed can not be neglected. Therefore the belt elements move relative to the conveyor, as does the real belt. This also implies that the bulk solid material mass does not move through the element grid but is fixed to an element as long as it is not discharged from the belt (head pulley).

It is beyond the scope of this paper to fully discuss the details of two-dimensional finite element model. Like introduced in section 2.3 they incorporate the mathematical description of multibody dynamics to easy for example the handling of the boundary conditions. Also see [1] or [46].

### 5 Application of the design parameters

To illustrate the application of the design parameters lets consider the start-up of a straight 2.5 km long overland conveyor. A start-up can be characterised by a start-up profile, which is the development of the belt speed as a function of time, and the start-up time, which is the time it takes to reach the stationary speed from rest. During start-up the transient belt tensions should be admissible (condition 1) and the dynamic behaviour of the belt should be acceptable (condition 2).

In [1] (summarized in [48]) a method is given to estimate the start-up time based on a limitation of the transient tension in the belt during start-up. Assume that  $V_b$  is the stationary belt speed,  $\dot{V}_b$  the acceleration of the belt (dV<sub>b</sub>/dt), C the ratio between the total resistance force including effects of accessories like side skirts and the main resistance force (DIN 22101), and f the effective friction coefficient (DIN 22101). If it is further assumed that g is the earth's gravitational acceleration and  $S_b$  the remaining safety factors during stationary operation, then the minimum required safety factor during start-up as a function of the belt's acceleration can be estimated as follows (also see [48]):



$$S_{\min} = S_{B} \left( 1 + \frac{\dot{V}_{b}}{C f g} \right)^{-1} \tag{7}$$

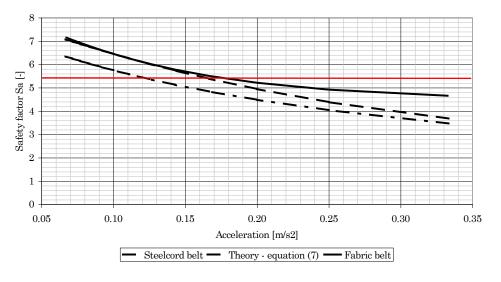
In equation (7) the effect of the elasticity of the belt is ignored. From equation (7) and assuming a constant acceleration ( $\dot{V}_b = \frac{V_b}{T_a}$ ), a start-up time can be determined as follows:

$$T_{A} = \frac{V_{b}}{C f g} \left( \frac{S_{A}}{S_{B} - S_{A}} \right) \tag{8}$$

In equation (8)  $S_a$  is the safety factor during transient operation. Note that the start-up time, defined by equation (8), does not explicitly depends on the conveyor length and that it is the same for an empty and a fully loaded belt! Both equation (7) and (8) do not account for the length of the conveyor system, except for the length dependency of DIN-22101 factor C. It is however doubtful that the relation between Din C and the conveyor length L is correct for application in the equations (7) and (8).

The dash-dotted line in Figure 9 shows the minimum required safety factor during start-up using equation (7) assuming  $S_b = 8$ , C = 1.04, f = 0.025 (very conservative), and g = 9.81. The solid line and the dashed line in Figure 9 show the safety factors for a steel cord belt and a fabric belt as found using the results of dynamic belt conveyor calculations. As can be seen in Figure 9, equation (7) gives a lower limit for the remaining safety factor in the belt as a function of the belt's acceleration. In order to determine an admissible acceleration for the belt, the required minimum safety factor during stationary operation  $S_a$  has to be taken into account. If it is assumed that the minimum value for  $S_a$  is 5.4 then the maximum value for the belt's acceleration is 0.12. This yields for this example a minimum start-up time of around 42 seconds assuming a stationary belt speed of 5 m/s.

### Minimum safety factors Smin versus belt acceleration



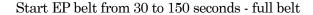
**Figure 9:** Remaining safety factor  $S_a$  as a function of the belt's acceleration.

Comparison of the safety factors obtained from the results of dynamic calculations of the start-up of a belt conveyor, where the effect of the belt's elasticity was taken into account, and as calculated with equation (7) shows that equation (7) gives a usable lower limit of the remaining safety factor. In other words: if one uses equation (8) to determine an appropriate start-up time then the transient belt tensions are admissible.



The drawback to using equation (8) for determining the belt's start-up time is that only the condition that the transient belt tensions have to admissible is met. It remains to be seen whether or not the belt's dynamic behaviour is acceptable.

The Figures 10 and 11 show the development of the belt speed as a function of the start-up time for so-called S starts see [1] or [48].



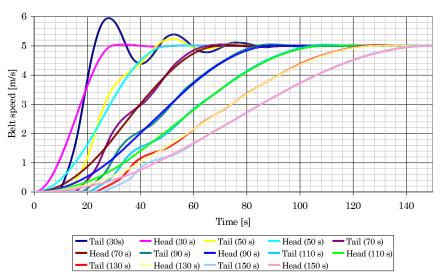
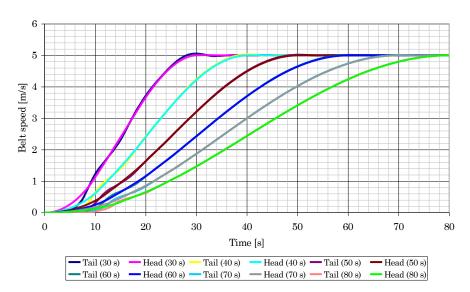


Figure 10: Belt speed as function of start-up time - EP belting.

Assume that the EP belt has a stiffness ( $E_bA_b$ ) of 5,880 kN and a belt weight per unit length of 15.2 kg/m. In that case the axial wave speed  $c_1$  can be calculated with equation (2) and is 621 m/s. With a load coupling factor of 0.73 the axial wave speed for a fully loaded belt is 455 m/s, which can be calculated with equation (5). Using  $c_1$  for a fully loaded belt the axial wave period, which is the time it takes a stress wave to travel through the whole belt and back, is 22 seconds. This wave period can clearly be recognized in Figure 10.

Further assume that the steel cord belt has a stiffness ( $E_bA_b$ ) of 47,642 kN and a belt weight per unit length of 23.2 kg/m. In that case the axial wave speed  $c_1$  for a steel cord belt is 1432 m/s. With a load coupling factor of 0.77 the axial wave speed for a fully loaded belt is 1098 m/s. Using  $c_1$  for a fully loaded belt the axial wave period is 9 seconds. This wave period can be recognized in Figure 11.





#### Start of Steelcord Belt from 30 to 80 seconds

Figure 11: Belt speed as function of start-up time – Steel cord belting.

If the Figures 10 and 11 are compared then it can be seen that there is much more belt speed overshoot visible in Figure 10 (EP belt) than in Figure 11 (St belt). If the belt's dynamic behaviour is found acceptable as soon as the overshoot is less than 1% of the stationary belt speed then 110 seconds and 50 seconds are acceptable start-up times for respectively the EP belt and the steel cord belt. For both belts this implies that the start-up is acceptable if the start-up time is more than 5 times the axial wave period. Note that the decay of the belt speed overshoot depends on the damping in the belt conveyor system.

Summarising: for the given example a start-up is acceptable if the start-up time is either calculated with equation (8) or taken as 5 times the axial wave period, whichever period of time is longer.

For more information on the practical verification of the results of dynamic analyses it is referred to [47].

### 6 Examples of dynamic simulations

As an example of the application of dynamic simulations in the design process of belt conveyors lets consider one of the belt conveyors of the Amplats Maandagshoek project, the North Shaft decline conveyor. This conveyor has a length of 1.35 km, a lift of 220 meter, and a capacity of 600 tons per hour. For this kind of belt conveyors the attention is focussed on the stop. Figure 12 shows the belt speed at the head pulley during a drift stop for various flywheel sizes. The inertia of the drive system is 9.5 kg/m² per motor. Without the application of a flywheel to increase the inertia of the drives/drive pulleys, the belt stops within4 seconds. It shows a significant reverse belt reaction after stopping and the belt tension at the tail is "zero", which in practice means that the belt drops onto the floor. Figure 13 shows that the minimum belt tension is acceptable if a large flywheel is used (100 kg/m²). In that case the belt speed during stopping is acceptable as well, see Figure 12. Unfortunately, application of a flywheel of this size results in long stopping times in case the belt is unloaded, see Figure 14.



## Normal stop fully loaded belt - no brake Belt speed at head pulley

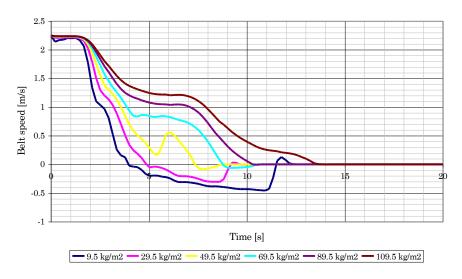
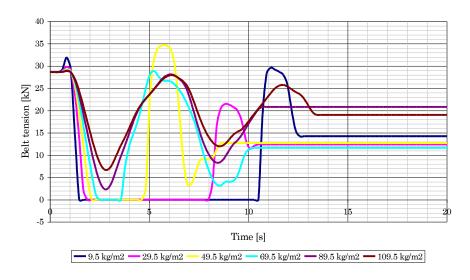


Figure 12: Belt speed during a drift stop of a fully loaded belt – various flywheel sizes.

### Normal stop fully loaded belt - no brake Minimum belt tensions



**Figure 13:** *Minimum belt tension during a drift stop of a fully loaded belt various flywheel sizes.* 



### Normal stop empty belt - no brake Belt speed at head pulley

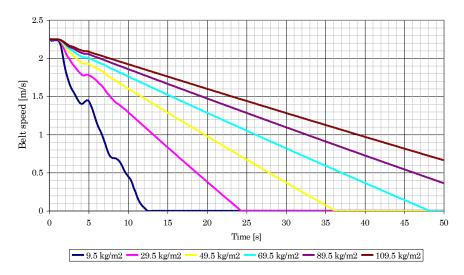


Figure 14: Belt speed during a drift stop of an empty belt – various flywheel sizes.

To limit the stopping time to about 30 seconds, which is also acceptable for emergency stopping, a brake is required. The question is however what brake size is necessary. Application of a brake, in this case on the pulley right after the take-up pulley at the head, also increases the belt tension during stopping and decreases the stopping time. The process of selecting a brake size and a flywheel size is called dynamic tuning. For this specific conveyor a 5 kNm brake and a flywheel of 60 kg/m² was an optimal combination. The stopping time for a fully loaded belt was 8 seconds as can be seen in Figure 15. Figure 17 shows that an empty belt stops in 31 seconds. Also the minimum belt tension is acceptable, see Figure 16.

# Normal stop fully loaded belt - 5 kNm brake Belt speed at head pulley

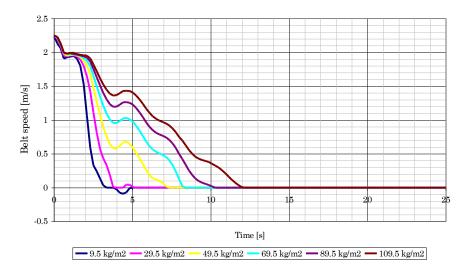
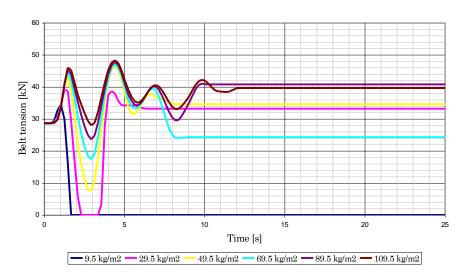


Figure 15: Belt speed during a braked stop of a fully loaded belt – various flywheel sizes.



### Normal stop fully loaded belt - 5 kNm brake Minimum belt tensions



**Figure 16:** *Minimum belt tension during a braked stop of a fully loaded belt various flywheel sizes.* 

### Normal stop empty belt - 5 kNm brake Belt speed at head pulley

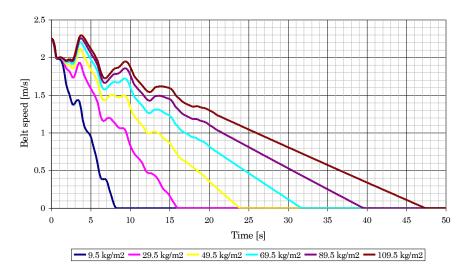


Figure 17: Belt speed during a braked stop of an empty belt – various flywheel sizes.

### 7 Frequently asked questions

With the examples shown in Section 5 and 6, the following frequently asked questions (FAQ's) can be answered:

1) What will happen if I change my steel cord belt for a fabric belt? If a steel cord belt is changed for a fabric belt then the start-up and stop procedures have to be changed. In particular the starting and stopping time have to be extended. In terms of the example



- used in Section 5 the start-up and stop time need to be multiplied by a factor 2.5 if a steel cord belt is changed for a fabric belt.
- What do we need to do if we want to double the belt speed of our conveyor? Besides of course changing the drive system, there are no requirements as far as steady state running is concerned assuming that the stationary speed is lower than about 8 m/s. For starting no changes are required in terms of starting time assuming that the start-up time originally was dynamically acceptable and as long as the start-up time is longer than the belt speed multiplied by a factor 8.3. As far as stopping is concerned, if a brake was originally used to stop the system than the brake capacity has to be quadrupled assuming that the stopping time may not be changed.
- 3) What can we do about low belt tensions during stopping? If the belt tension during stopping is too low then there are a number of options. A flywheel can be used to extend the stopping time, which delays the decrease of the belt tension. Sometimes the belt is stopped before the belt tension reaches unacceptably low levels. If the stopping time is too long, a brake can be used to increase the belt tension or to decrease the stopping times. Alternatively, the belt's pretension can be increased by increasing the take-up mass or a capstan, which is a brake in the take-up system, can be used to increase the belt tension during stopping.
- 4) What will happen if "high-tech" low indentation rolling resistance rubber is applied? Low indentation rolling resistance rubber means that less energy is required to overcome the rolling resistance of a belt conveyor and therefore to run the conveyor at steady state. During starting however, the same amount of energy is required to accelerate the belt, the rolls and the bulk material. Therefore as far as the starting procedure is concerned no changes are required. If a brake was used to stop the conveyor then no changes are required. If the conveyor drifted to rest then the stopping time will increase. This may give raise to installing a brake to ensure that the belt in case of an emergency stops in an appropriate time.

### 8 Conclusion

This paper gave an overview of the work done on the mathematical description of dynamics of belt conveyor systems from the early fifties till date. It showed that the insight gained into the dynamics of belt conveyors, easy to use equations, and the significant improvement of drive technology over the last twenty years, enabled the design of start and stop procedures for belts with "easy geometries" without using dynamic simulations. For many belt conveyors however, the analysis of the non-stationary operation still requires the application of dynamic simulations, in particular when analysing emergency stops and when determining "what if" scenarios. With the availability of dynamic simulation packages today virtual prototyping of large belt conveyor systems has become a reality!

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