



## **BELTCON 4**

Criteria for the Optimum Design of Drive & Brake  
Units in Belt Conveyors

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# CRITERIA FOR THE OPTIMUM DESIGN OF DRIVE AND BRAKE UNITS IN BELT CONVEYORS

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## SUMMARY

Introductorily a method to calculate the quasi-steady acceleration processes of belt conveyors during starting and braking is developed. Main parameters for an analysis of these operational phases are the corresponding accelerations and the total of peripheral forces of driven or braked pulleys. Relating the latter to the resistances to motion, the force ratios  $p_A$  and  $p_B$  for the characterization of starting and braking processes are defined. These parameters, whose interaction with the characteristics of drive and brake units is analyzed, and the product  $C \cdot f$ , characterizing the frictional resistances to motion of belt conveyors, determines the acceleration behaviour during starting and braking. Inclined conveyors are additionally characterized by the value of the quotient  $\sin \delta / (C \cdot f)$  and in this way by the slope angle  $\delta$ . Regarding e.g. steeply inclined belt conveyors under load, this quotient is the reason for the fact that the starting acceleration of uphill conveyors and the braking deceleration of downhill conveyors are considerably higher than in any comparable horizontal conveyor with the same power required and the same limitation of the driven or braked pulleys' peripheral forces in these transient operating conditions and thereby with the same force ratios  $p_A$  and  $p_B$ . Basing on the introduced relations which describe the dynamic behaviour of belt conveyors in their acceleration phases during starting and braking, criteria for optimum values of the corresponding force ratios  $p_A$  and  $p_B$ , the starting acceleration  $a_A$  and the stopping deceleration  $a_B$  are developed.

## 1. Introduction

In the last 30 years belt conveyor systems have been developed into means of transport for bulk materials with mass flows up to 37 500 t/h, with centres of pulleys with more than 13 000 m and with belt speeds up to 7.5 m/s. As a result of this development belt conveyors were installed with power requirements up to 12 000 kW in the brown coal mines of Rheinbraun in the Federal Republic of Germany.

In the past the main aspect in designing large-capacity and overland belt conveyors was the power requirements in their steady operating conditions. After the basis for a sufficiently accurate calculation of the power requirement of belt conveyors had been created, last not least by the results of investigations at the University of Hannover (e.g. /1, 2, 3, 4/), the dynamic behaviour of belt conveyors in their "non-steady", i.e. transient operating conditions when starting and braking was increasingly considered, especially with respect to a more accurate prediction of the transient forces and thus for an optimum design of the conveyor belt, in general the most expensive component of belt conveyor systems (e.g. /5, 6, 7, 8, 9, 10/).

At the beginning of this contribution which deals with some essential aspects which should be considered when designing drive and brake units with regard to the transient operating conditions of belt conveyors a method shall be introduced making it possible to optimize these operating conditions with regard to belt stress and their duration in those cases where the characteristics of drive and brake units permit the operating conditions mentioned to be considered "quasi-steady", i.e. "steady-like". As demonstrated in a different paper, this assumption is generally allowed when the drive and brake units have limited rates of torque rise. Only in this case the peripheral forces of the driven or braked pulleys, the local tensions in the belt and its acceleration and deceleration can be calculated in an easy but correct way when idealizing the belt as a rigid body having the same local velocity on its entire length. When the acceleration and the deceleration of the belt is determined, it is rather simple to calculate the total local belt forces by superimposing initial forces, resistances to motion as well as acceleration and deceleration forces in the upper and lower strand.

## 2. Fundamentals for an Analysis of the Quasi-steady Operating Conditions of Belt Conveyors

### 2.1 Fundamental Equations to Calculate Belt Acceleration and Deceleration

The basis for the calculation of belt accelerations and decelerations in quasi-steady operating conditions of belt conveyors is the following equation which is applicable for starting and braking phases and bases on the assumption that the resistances to motion in these operating conditions are approximately the same as the resistances  $F$  in the steady operating conditions /11/:

$$F_x = a_x \cdot m + F \quad (1)$$

where:

$F_x$ = total of peripheral forces of driven or braked pulleys  $a_x$ = belt acceleration starting: $a_A > 0$ braking : $a_B < 0$ (deceleration)	}	during quasi-steady operating conditions of belt conveyor (subscript x: $x = A$ : starting $x = B$ : braking)
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$m$  = total of translatorily and rotatorily moved masses (the latter reduced to their periphery) without directly driven or braked components of belt conveyor

$F$  = total of the resistances to motion

In general, the totals of masses  $m$  and the resistances to motion  $F$  can be calculated easily and therefore equation (1) can simply be applied. Generalized statements with respect to the interaction between the total of peripheral forces  $F_x$  and the belt acceleration  $a_x$  are possible, if at first the force  $F_x$  is related to the total of the resistances to motion  $F$  and if thus the force ratio  $p_x$  is introduced:

$$\frac{F_x}{F} = p_x = 1 + a_x \cdot \frac{m}{F} \quad (2)$$

In addition, it is advisable to introduce the "natural acceleration"  $a_n$  of a belt conveyor which is expressed through the ratio  $F/m$ . This is a fictive acceleration of all moved masses  $m$  in the upper and lower strand if these are accelerated by the force  $F$ . Thus the following fundamental relationship between the force ratio  $p_x$  and the acceleration  $a_x$  is obtained:

$$p_x = 1 + \frac{a_x}{a_n} \quad (3)$$

Every belt conveyor is characterized by a certain ratio  $m/F$  and thus by a certain value of the acceleration  $a_n$ , so that one of the parameters  $p_x$  or  $a_x$  can be calculated if the other is numerically known. Whereas equation (3) is used if the ratio  $p_x$  is to be calculated, the following equation is applicable for the determination of the acceleration  $a_x$ :

$$a_x = (p_x - 1) \cdot a_n \quad (4)$$

The equations (3) and (4) are applicable for the acceleration processes during the starting (force ratio  $p_A$ , acceleration  $a_A$ ) and the braking of belt conveyors (force ratio  $p_B$ , deceleration  $a_B$ ). As demonstrated in the following, these equations are very important for an optimization of these processes with regard to minimum transient belt forces, minimum thermal load on drive and brake units and minimum duration. Before descriptively dealing with these aspects, some reference shall be given to the value of the natural acceleration  $a_n$ , and the force ratios  $p_A$  and  $p_B$  shall be expressed by the parameters of the drive and brake units.

On condition that the belt conveyors, which shall be investigated with respect to their non-steady operating conditions, have a steady inclination and load distribution on their entire length, but no special resistances due to idler tilting, discharge plows and friction between load and skirtboards outside the loading points, the following defining equation for the acceleration  $a_n$  can be deduced (cf. /11/):

$$a_n = a_{no} \cdot \beta \quad (5)$$

with

$$a_{no} = C \cdot f \cdot g \cdot c_m \quad (6)$$

$$\beta = 1 + \frac{\sin \delta}{C \cdot f} \cdot \eta \quad (7)$$

for  $\cos \delta \approx 1$ .

The following parameters apply:

- $c_m$  = mass ratio  
(ratio of the masses, being in relation with the frictional resistances to motion, to the masses  $m$  according to equation (1))
- $C \cdot f$  = parameter product (according to the standard DIN 22101), which characterizes the main and secondary resistances of belt conveyors in the upper and lower strand
- $g$  = gravitational acceleration ( $g = 9.81 \text{ m/s}^2$ )
- $\beta$  = acceleration ratio
- $\delta$  = average slope angle of belt conveyor
- $\eta$  = ratio of the masses due to load to the total mass in upper and lower strand, being in relation with the frictional resistances to motion.

Equation (6) defines the fictive acceleration  $a_{no}$  of all moved masses  $m$  in the upper and lower strand, if these are accelerated by a force equivalent to the total of all those resistances to motion, which are conditioned by friction. The influence of the slope of a conveyor and thereby the influence of its slope resistance, is taken into account by the acceleration ratio  $\beta$ .

Of those parameters which determine the value of the acceleration  $a_{no}$  the product  $C \cdot f$  is the most important. Belt conveyor systems which have centres of pulleys with more than 500 m and which are designed and operated in accordance with the relevant standards are generally characterized by the value of the product  $C \cdot f$  in the range  $0.015 \leq C \cdot f \leq 0.030$ , if they are operated under the climate conditions of central Europe. With the parameter  $c_m$  which is

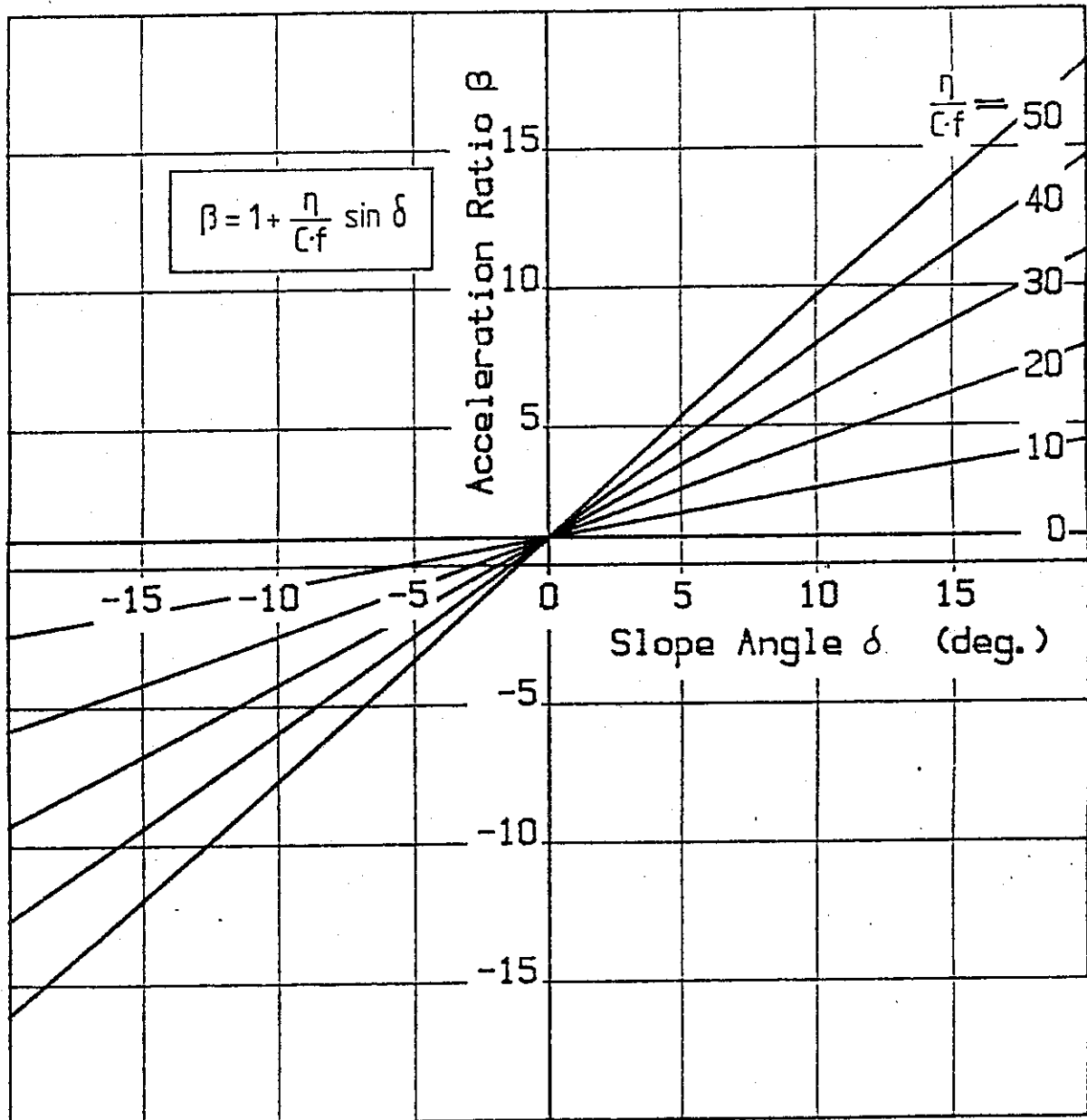


Figure 1 Acceleration ratio  $\beta$  versus slope angle  $\delta$  of belt conveyors  
( $\delta < 0$  : downhill conveyors;  $\delta > 0$  : uphill conveyors)

generally slightly less than 1, and with the gravitational acceleration  $9.81 \text{ m/s}^2$  the range

$$0.15 \text{ m/s}^2 \leq a_{no} \leq 0.30 \text{ m/s}^2 \quad (8a)$$

can be calculated for the acceleration  $a_{no}$ . For belt conveyors with centres of pulleys above 500 m the following average value can be used as approximate basis for calculation:

$$a_{no} \approx 0.2 \text{ m/s}^2 \quad (8b)$$

According to equations (4), (5) and (7) the acceleration ratio  $\beta$  is to be regarded as a factor by which the acceleration or deceleration of inclined belt conveyors ( $\delta \neq 0$ ) in their non-steady operating conditions is increased in comparison with horizontal belt conveyors ( $\delta = 0$ ) with the same force ratios  $p_A$  and  $p_B$ , especially with the same values of resistances to motion  $F$  and the same peripheral forces  $F_A$  or  $F_B$  of the driven or braked pulleys. The ratio  $\beta$  is essentially determined by the slope angle  $\delta$  and the mass ratio  $n$  and it varies over a wide range:

The quotient  $\eta/(C \cdot f)$  is usually in the range  $0 \leq \eta/(C \cdot f) \leq 50$ . The lower limiting value is characteristic of unloaded belt conveyors, and the upper one of loaded belt conveyors with low frictional resistances to motion and high values of the mass ratio  $n$ . Usual slope angles  $\delta$  between  $-18^\circ$  and  $+18^\circ$  yield acceleration ratios in the range  $-14.5 \leq \beta \leq 16.5$  (cf. Figure 1). As a consequence, steeply inclined belt conveyors with nominal load and angles  $\delta$  in the range  $\pm 18^\circ$  are characterized by 15 or 16 times higher accelerations than horizontal belt conveyors with the same resistances  $F$  and peripheral forces  $F_x$  and thereby the same parameters  $p_x$ . As a result it can be concluded that the ratios  $p_x$  and thereby the pulley forces  $F_x$  cannot be deduced by observing the acceleration processes in the starting and braking phases, as it is often done in practice.

Completely different results will be found when calculating the parameter  $\beta$  for slightly inclined belt conveyors. Analyzing e.g. a downhill conveyor with a slope angle  $\delta$  between  $-1^\circ$  and  $-1.5^\circ$ , additionally characterized by  $\sin \delta = -C \cdot f$  and the mass ratio  $\eta = 0.65$ , the equation (7) yields  $\beta = 0.35$ . Thus this belt conveyor is accelerated or decelerated approximately with one



third of those values, which are typical for a horizontal conveyor with the same force ratio  $p_x$ . Before discussing further consequences, the force ratios  $p_A$  and  $p_B$  shall be expressed by the parameters of the drive and brake units.

## 2.2 Relationship between the Force Ratios $p_A$ and $p_B$ and the Parameters of the Drive and Brake Units

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The accelerating and decelerating effect of drive and brake units when starting and braking, and therefore also the values of the force ratios  $p_A$  and  $p_B$  are determined by the characteristics and rotating masses of these units and by the efficiency of their reduction gears. With regard to the following analysis of the starting processes of slightly inclined, of horizontal and of steep uphill belt conveyors and also to the analysis of braking processes of steep downhill belt conveyors, the force ratios  $p_A$  and  $p_B$  shall only be determined for these types of belt conveyors. Characteristic of both is that the power requirement of the loaded belt is the criterion for the design of the drive and brake units of these conveyors.

The force ratio  $p_x$  (starting:  $x = A$ ; braking  $x = B$ ) depends on the corresponding torque ratios  $p_{x0}$ , deduced from the characteristics of the drive and brake units and related to the total of nominal torque of all drive units, in the following way:

$$p_x = 1 + (R \cdot p_{x0} - 1) \cdot \left(1 - \frac{m_{Ant}}{\Sigma m_x}\right) \quad (9)$$

Thus the equation for  $p_{x0}$ :

$$p_{x0} = \frac{1}{R} \cdot \left(1 + \frac{p_x - 1}{1 - \frac{m_{Ant}}{\Sigma m_x}}\right) \quad (10)$$

where:

$m_{Ant}$  = total of effective masses of rotating parts of the drive and brake units reduced to the periphery of the driven and braked pulleys.

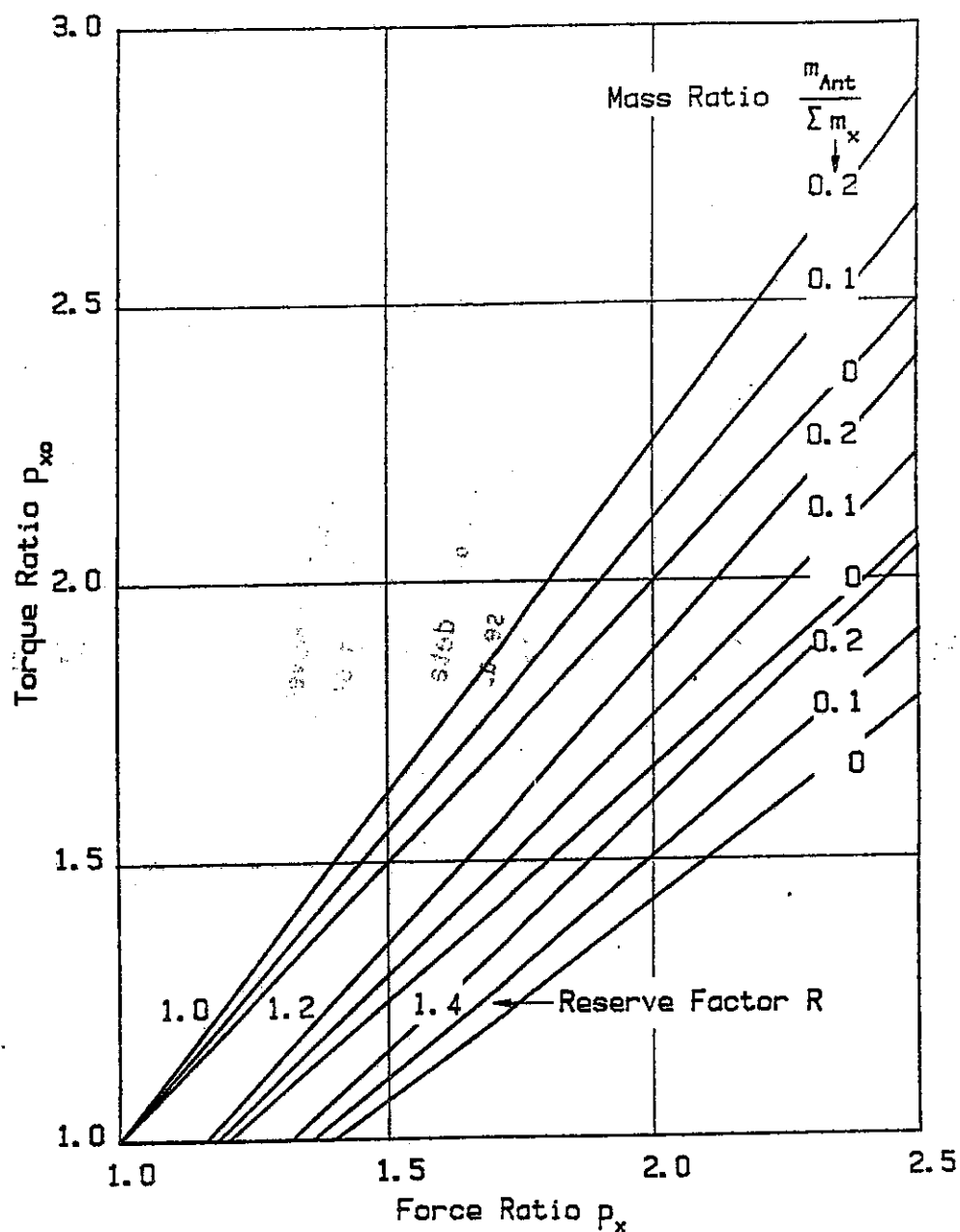


Figure 2 Relationship between the torque ratio  $p_{x0}$ , depending on the characteristics of drive and brake units, and the force ratio  $p_x$ , characterizing the pulleys' peripheral forces during quasi-steady acceleration phases, with the reserve factor  $R$  and the mass ratio  $\frac{m_{Ant}}{\Sigma m_x}$  as parameters of loaded belt conveyors

$$\Sigma m_x = m_{Ant} + m \cdot r^y$$

$$x = A : y = -1; x = B : y = +1$$

(definition of the mass  $m$  by equation (1))

$R$  : reserve factor

(ratio of the total of nominal torque of all drive units to the total of required torque of the drives in the steady operating conditions of a belt conveyor)

Figure 2 demonstrates graphically the relationship between the parameters  $p_x$  and  $p_{x0}$  for belt conveyors with mass flows in the range of their nominal load. It is obvious that, if  $R = 1$ , for all values of the ratio  $m_{Ant}/\Sigma m_x$  the parameter  $p_{x0}$  is always higher than the force ratio  $p_x$ . If  $R > 1$ ,  $p_{x0}$  is lower than  $p_x$ . This the more, the lower the ratio  $m_{Ant}/\Sigma m_x$  is. This has to be considered especially when belt conveyors are operated with partial load or unloaded, in which cases the reserve factor increases to  $R > 3$  and the mass ratio to  $m_{Ant}/\Sigma m_x > 0.5$ . Summing up, it is essential to differentiate between the parameters  $p_x$  and  $p_{x0}$ . Only by taking this into consideration unproductive discussions between the experts for the drive and brake units and those for the belt conveyors about the optimum values of "starting-factors" and "braking-factors" - frequently used synonyms for the force ratios  $p_x$  and  $p_{x0}$  - can be avoided.

The preceding demonstration explains the mutual dependence between the torque ratio  $p_{x0}$ , essentially determined by the characteristics of the drive and brake units, and the force ratio  $p_x$ , which strongly influences the peripheral forces of the driven or braked pulleys during the quasi-steady operating conditions of belt conveyors. In the following, the influences of the ratio  $p_x$  on the acceleration and deceleration processes of starting and braking belt conveyors shall be demonstrated.

### 3. Criteria for Optimum Values of the Force Ratio $p_x$ and the Acceleration $a_x$ during Starting and Braking of Belt Conveyors

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The ratio of peripheral forces  $p_x$  is determined by the characteristics of the drive and brake units, the resistances to motion of the conveyor and, in addition to this, by the total amount and by the distribution of the translatorily and rotatorily moved masses of a belt conveyor installation. When designing and calculating belt conveyors, in general certain values of the parameter  $p_x$  are assumed, and basing on them, the characteristics of drive and brake units are determined and realized by an expedient selection of their components. These assumptions with respect to force ratio  $p_x$  shall be analyzed in the following with regard to the acceleration and deceleration processes in belt conveyor systems, if they are quasi-steady.

#### 3.1 Starting of Belt Conveyors

In order to minimize the belt stress, it is generally necessary to limit the total pulley peripheral force  $F_A$  in the starting phases. According to the design of drive units, the frequency and duration of the starting process has to be considered additionally. For an optimum design of the force ratio  $p_A$  and of the starting acceleration  $a_A$  of long and high-capacity belt conveyor systems the following criteria should usually be taken into account (cf. recommendations in the standard DIN 22101):

- When starting the maximum pulley peripheral force  $F_{Amax}$  should not exceed  $F_{Amax} = 1.3$  to  $1.7$  times the total resistances to motion  $F_{max}$  under the most unfavourable conditions (loading condition, distribution of load).
- In order to accelerate the masses in the upper and lower strand there should, however, be an acceleration force available under the most unfavourable starting conditions which amounts at least to 20 % of the total frictional resistances to be taken into account in this case, and which enables the system to run up to full operating speed within a maximum permissible time with respect to the thermal stress of the drive units.
- The force  $F_A$  must be selected in such a way that the frictional contact

between the load and the belt is ensured for the starting acceleration  $a_A$  corresponding with the force  $F_A$ .

These criteria shall now be analyzed with regard to the starting process of slightly inclined, horizontal and steep uphill belt conveyors, which are characterized by  $\sin \delta > -C \cdot f$  and therefore by the power requirement of the loaded belt being the criterion for the design of the drive units.

The first criterion indicates maximum values for the force ratio  $p_A$  in the range  $1.3 \leq p_{Amax1} \leq 1.7$ , which are according to equations (4), (5), (6), corresponding with the maximum starting acceleration  $a_{Amax1}$ , if the natural acceleration  $a_{no}$  and the acceleration ratio  $\beta$  is given:

$$a_{Amax1} = (p_{Amax1} - 1) \cdot a_{no} \cdot \beta \quad (11)$$

Combining the upper and lower limiting values of the parameters  $p_{Amax1}$  with those of the acceleration  $a_{no}$  according to equation (8a) the following result is obtained for the starting acceleration  $a_{Ao}$  in the case of horizontal belt conveyors ( $\beta = 1$ ):

$$0.045 \text{ m/s}^2 \leq a_{Ao} \leq 0.21 \text{ m/s}^2.$$

The mean acceleration  $a_{Ao} = 0.1 \text{ m/s}^2$ , which results from the mean maximum force ratio  $p_{Amax1} = 1.5$  and the mean acceleration  $a_{no} = 0.2 \text{ m/s}^2$  (cf. equation (8b)), corresponds with the experiences of the usual design of belt conveyor systems. Assuming the acceleration  $a_{Ao} = 0.1 \text{ m/s}^2$  to be constant during the entire starting process, the belt velocity 5 m/s will be reached after a starting time of 50 seconds.

Assuming a certain belt velocity of the operating conditions to be constant higher force ratio  $p_A$  has to be realized in order to limit the starting time  $t_A$  with regard to the thermal stress of the drive units generally in the case of long horizontal conveyors with low  $C \cdot f$ -values and especially in long slightly inclined belt conveyors with  $\sin \delta > -C \cdot f$  and acceleration ratios  $\beta$  in the range  $1 - \eta \leq \beta \leq 1$ . For the practical determination of the parameters  $p_A$  and  $a_A$  on the basis of the equations (4), (5), (6), (7), the diagram in figure 3 can be used. While the upper diagram refers to horizontal and slightly inclined downhill belt conveyors, the lower one refers to steep

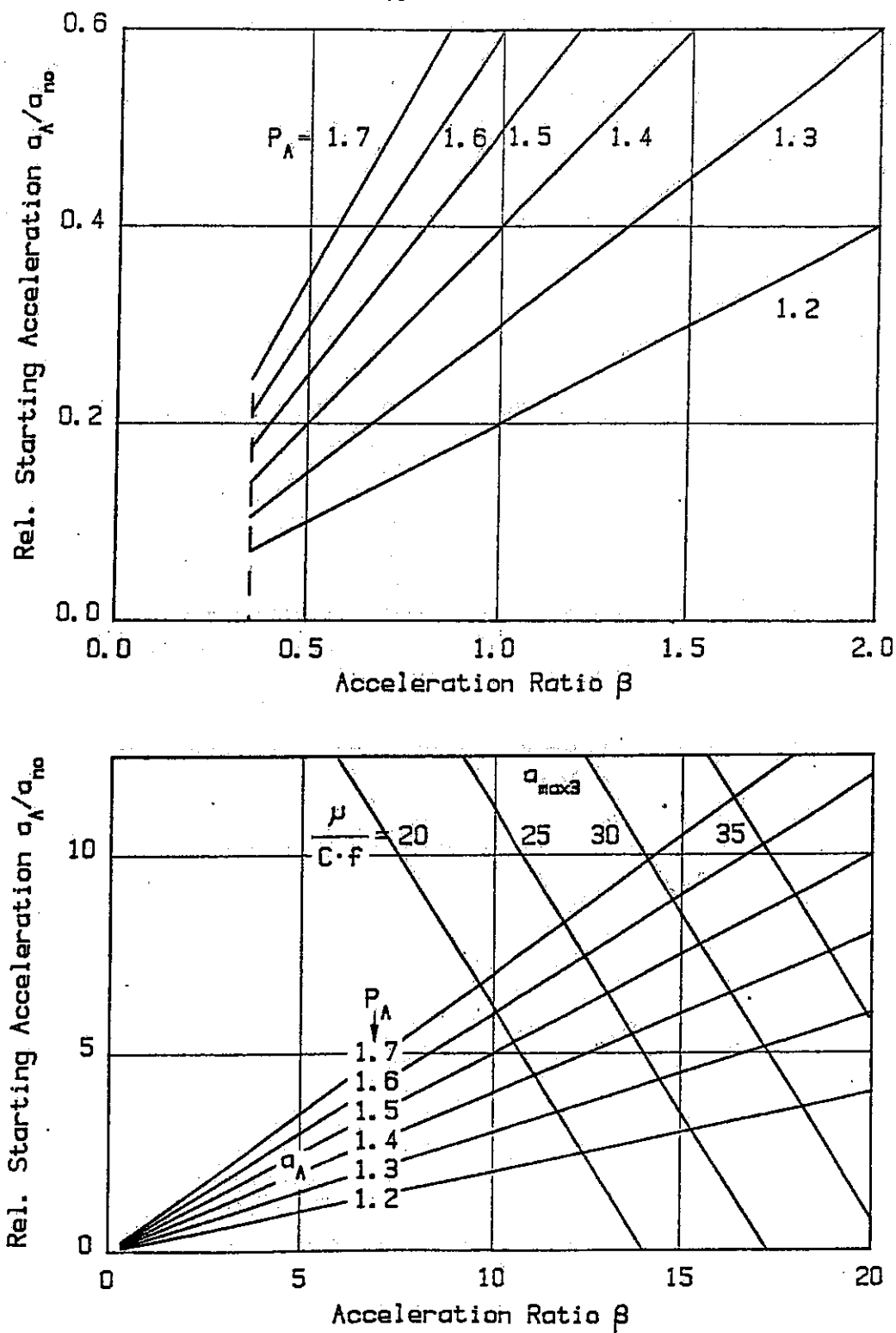


Figure 3 Diagram for the determination of the starting acceleration  $a_A$  as a function of the force ratio  $p_A$  and the acceleration ratio  $\beta$  (lower limiting value  $\beta_{min} = 0.35$  according to equation (7) for  $\sin \delta = -C \cdot f$  and  $\eta = 0.65$ ).

- a. horizontal and slightly inclined belt conveyors  
(for  $\sin \delta > -C \cdot f$ )
- b. steep uphill belt conveyors  
(maximum acceleration  $a_{Amax3}$  calculated for loaded belt conveyors with  $\eta = 0.65$ )

uphill belt conveyors.

The second criterion indicates a certain minimum value for the acceleration force  $a_A \cdot m$ , which should be at least 20 % of all those resistances to motion which are caused by friction. This criterion corresponds with the demand to realize a minimum acceleration of 20 % of the fictive acceleration  $a_{no}$ :

$$a_{Amin2} = 0.2 \cdot a_{no} \quad (12)$$

As a consequence of this and with regard to the values of the parameter  $a_{no}$  for belt conveyors with centres of pulleys above 500 m (cf. equations (8a), (8b)), these conveyors should have a minimum acceleration during starting which is independent from their slope and lies in the range of

$$0.03 \text{ m/s}^2 \leq a_{Amin2} \leq 0.06 \text{ m/s}^2.$$

This means that belt conveyor systems designed in this way have maximum starting times in the range

$$167 \text{ s} \geq t_{Amax2} \geq 83 \text{ s},$$

if their operational velocity is 5 m/s. As a result of this it has to be concluded that 20 % of the frictional resistances to motion as accelerating force for the masses in upper and lower strand and thus 20 % of the acceleration  $a_{no}$  should generally be considered as absolute lowest limiting values with regard to the thermal stress of the drive units. This refers to slip-ring induction motors with resistance starters as well as to squirrel cage induction motors with those hydrodynamic couplings, where the coupling-liquid flows through an external circuit in order to cool the liquid and to control or regulate the torque-transmission. Especially for conveyors with low values  $a_{no}$  and directly driving squirrel cage induction motors, as well as such with drive units with uncontrolled or non-regulated slipping starting-couplings, it would be advantageous to start them with higher minimum acceleration forces and thus minimum accelerations in order to limit the slip-heat during the starting phases. With regard to the heat-absorption capacity and heat-abstracting characteristic of drive units and the starting frequencies of belt conveyor systems, the minimum accelerations  $a_{Amin}$  will generally be in the range of about  $0.4 \cdot a_{no}$ .

The third criterion limits, especially for steep uphill conveyors ( $\sin \delta > 0$ ), the starting acceleration with regard to the frictional contact between load and belt. If fine-grained bulk material is conveyed, the following equation has to be considered (cf. standard DIN 22101):

$$a_{Amax3} = (\mu \cdot \cos \delta - \sin \delta) \cdot g \quad (13a)$$

$\mu$  = friction factor between load and belt

( $\delta < 0$  : downhill belt conveyors,  $\delta > 0$  : uphill belt conveyors)

When this formula is converted in the following way:

$$a_{Amax3} = \left( \frac{\mu \cdot \cos \delta}{C \cdot f \cdot c_m} - \frac{B - 1}{c_m \cdot \eta} \right) \cdot a_{no} \quad (13b)$$

it can be compared with the equations (11) and (12). Equation (13b) with the acceleration  $a_{Amax3}$  related to the parameter  $a_{no}$  and thus defined as the "relative" (abbr.: rel.) starting acceleration is graphically represented in figure 3, simplifying  $c_m \approx \cos \delta \approx 1$  and assuming high acceleration ratios  $B$ . It is obvious that the frictional contact is only endangered at acceleration ratios  $B > 14$ , if assuming the force ratio  $p_{Amax1} = 1.7$  and the quotient  $\mu/(C \cdot f) = 30$ , the latter representing normal conditions. According to figure 1 the ratio  $B = 14$  is corresponding with slope angles  $\delta > 19^\circ$ , assuming the quotient  $\eta/(C \cdot f) = 40$ . Thereby it can be concluded that normal belt conveyors with slope angles less than  $18^\circ$  are endangered with regard to the frictional contact between load and belt only in those cases where lower values of  $\mu/(C \cdot f)$  and higher values of  $p_A$  occur simultaneously. - In addition to this it shall, however, be emphasized that the figures given with respect to the permissible acceleration  $a_{Amax3}$  only apply to belt conveyors under nominal load with the mass ratio  $\mu = 0,65$ . For partially loaded or empty belt conveyors special calculations are necessary.

While the preceding explanations deal with the acceleration phases of starting belt conveyors with special consideration of their duration, in the following the value of the force ratio  $p_A$ , determining the peripheral forces  $F_A$  of the driven pulleys and thus the belt tensions during starting, shall be discussed. For that purpose the ratio  $p_A$  is, according to equations (3) and (5), expressed by the relative starting acceleration  $a_A/a_{no}$  and the



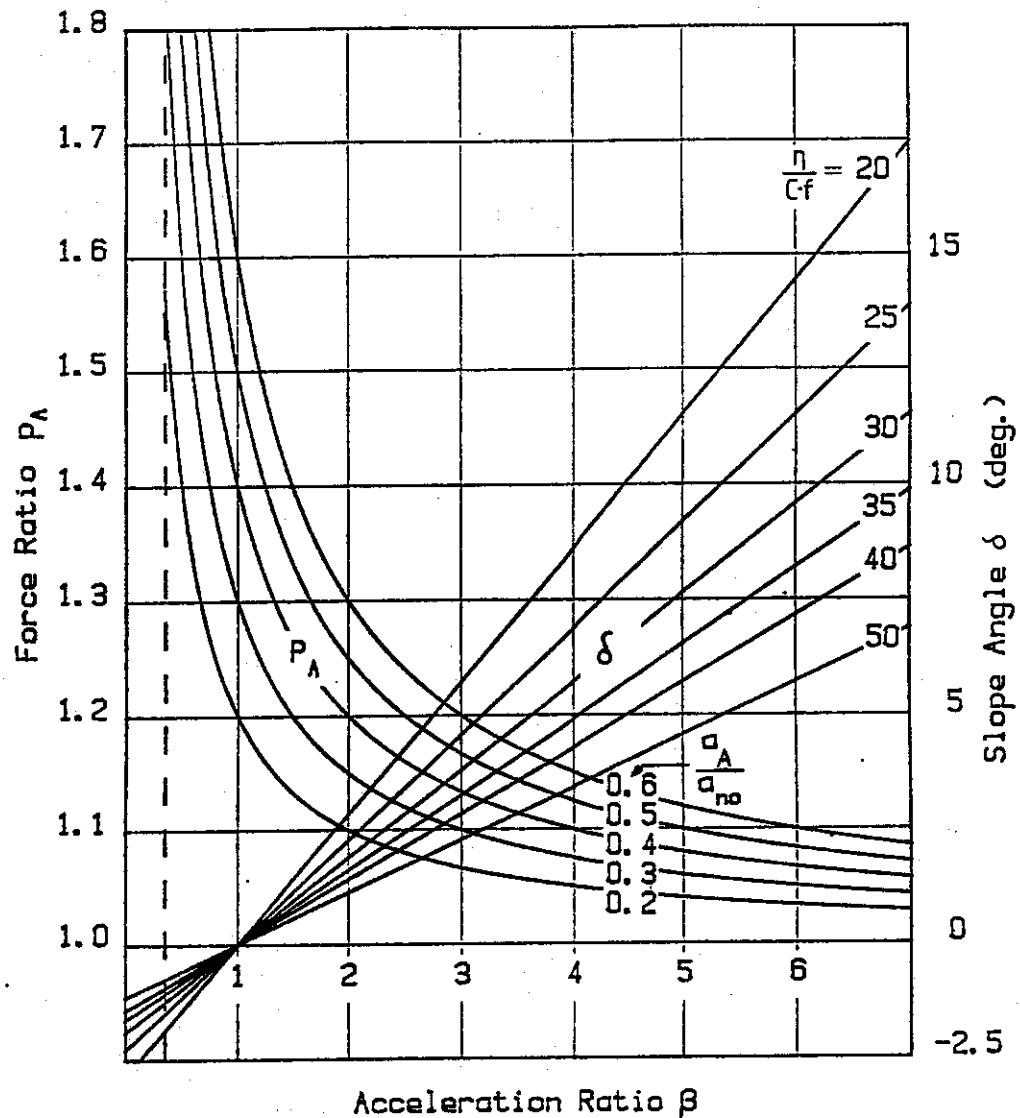


Figure 4 Diagram for the determination of the force ratio  $p_A$  as a function of the starting acceleration  $a_A$  and the acceleration ratio  $\beta$  for horizontal, slightly inclined and steep uphill belt conveyors  
(lower limiting value  $\beta_{min} = 0.35$  according to equation (7) for  $\sin \delta = -C \cdot f$  and  $\eta = 0.65$ )

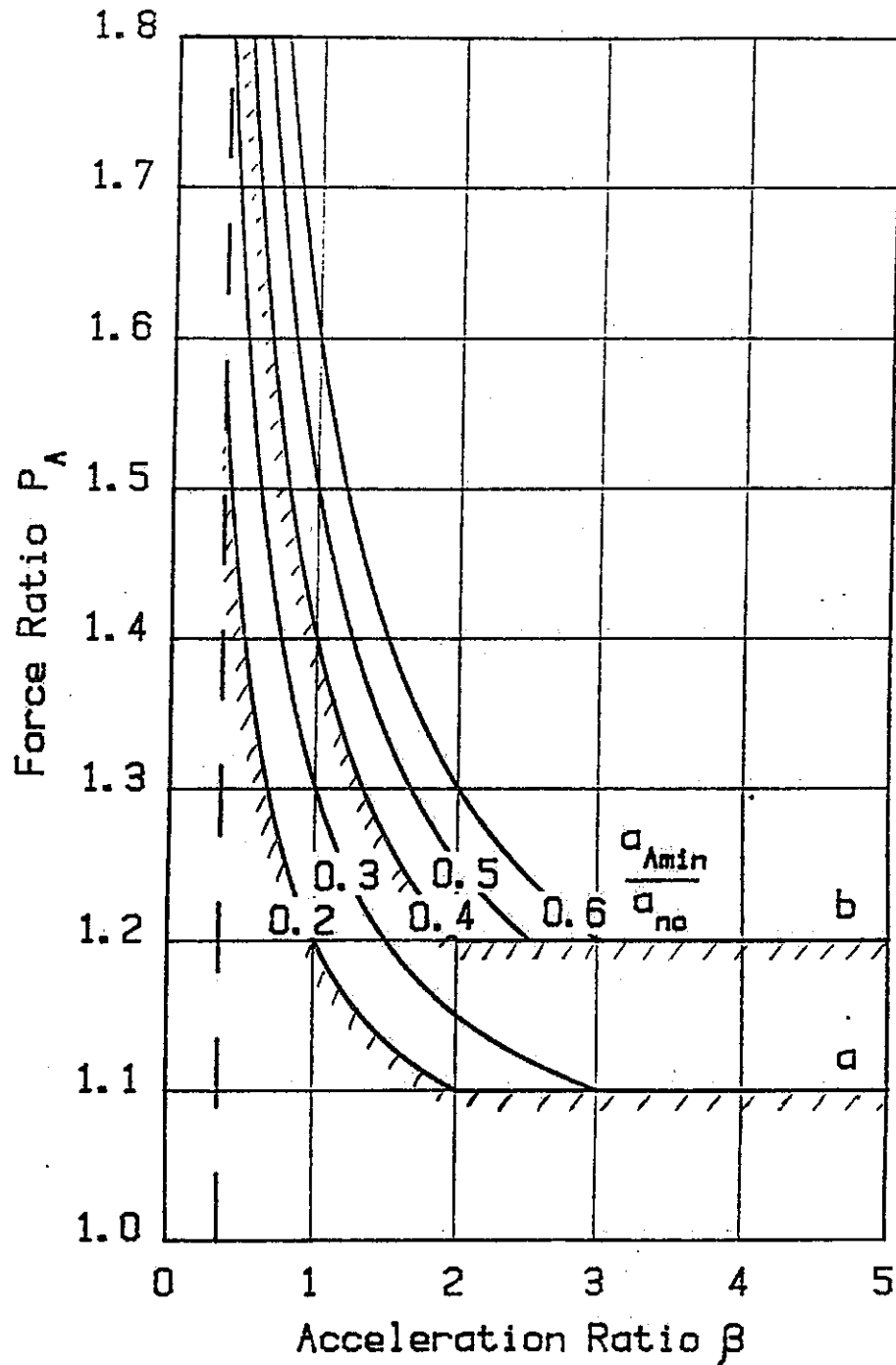


Figure 5 Diagram for the optimization of the force ratio  $p_A$  and the starting acceleration  $a_A$  for horizontal, slightly inclined and steep uphill belt conveyors  
(lower limiting value  $\beta_{min} = 0.35$  according to equation (7) for  $\sin \delta = -C \cdot f$  and  $\eta = 0.65$ )

acceleration ratio  $\beta$ :

$$p_A = 1 + \frac{a_A}{a_{no} \cdot \beta} \quad (14)$$

This relation is graphically represented in figure 4, together with the relationship between the parameter  $\beta$  and the slope angle  $\delta$ , for slightly inclined, horizontal and steep uphill belt conveyors with  $\sin \delta > -C \cdot f$  and thus  $\beta > 1 - \eta$ .

Figure 4 shows that only slightly inclined downhill belt conveyors with  $\beta < 1$  have minimum force ratios, determined by the minimum acceleration  $a_{Amin2}$ , with values in the range  $p_{Amin2} > 1.2$  and therefore approximately remain within the dimensions as stated by the first criterion for maximum values  $p_{Amax1}$ . Therefore these slightly inclined downhill belt conveyors have to be started with relatively high minimum force ratios  $p_A$  in order to achieve a sufficient limitation of the starting time and thereby of the thermal stress of the drive units.

For horizontal, especially uphill conveyors, however, lower figures of the parameter  $p_A$  are principally permissible with regard to the minimum acceleration  $a_{Amin2}$ , but not lower than those given for a minimum starting acceleration and thus a maximum starting time as determined by the thermal stress of the driving units. Force ratio  $p_A \leq 1.1$  for steep uphill conveyors, designed and operated in the usual way, should only exist in theory. In order to prevent any difficulties during starting, as a consequence of higher resistances to motion which exceed those used for the design of the drive units, even in the case of evenly loaded conveyors and with regard to their load controlled conveyors the force ratio  $p_{Amin} = 1.1$  should generally be considered as an absolute lower limiting value. When regarding, however, belt conveyors with uncontrolled and non-regulated drive units and uneven non-controlled loading, the minimum limiting value for  $p_A$  has to be increased at any rate. A minimum force ratio  $p_{Amin} = 1.2$  should be justified as a suitable limit /11/.

Figure 5 shows a diagram to optimize the force and acceleration ratios  $p_A$  and  $\beta$ . It is based on figure 4 and takes into account the requirements for driving units as specified above. The selection of ratios  $p_A$  less than the

minimum values  $p_{Amin}$ , defined by the limiting curves a and b, should only be allowed in justifiable, exceptional cases.

The higher limiting curve b, determined by the minimum starting acceleration  $a_{Amin} = 0,4 \cdot a_{no}$  and the minimum force ratio  $p_{Amin} = 1.2$  applies to most of the belt conveyors with uneven loading and to such with uncontrolled and non-regulated drive units during the starting phases. - On the other hand the lower limiting curve a, determined by the minimum starting acceleration  $a_{Amin} = 0.2 \cdot a_{no}$  and the minimum force ratio  $p_{Amin} = 1.1$ , only applies to belt conveyors with controlled loading and such drive units whose torque during starting is controlled or regulated depending on the load of the conveyor and whose thermal stressability is sufficient.

The practical design of drive units by optimizing the force ratio  $p_A$  with regard to minimum belt stress and permissible thermal stress of the driving units has additionally to take into consideration that the drive unit's torque during starting generally shows peaks and valleys. In these cases optimally designed units should under all conditions provide force ratios  $p_A$ , which remain above the discussed limiting curves during the total starting phases.

### 3.2 Braking of Belt Conveyors

The operation of belt conveyor systems requires brake units for the purpose of stopping the moving masses, and/or holding devices for the purpose of holding inclined installations stationary under load. For an optimum design of brake units the required total braking force  $F_B$  on the braked pulleys, the number and arrangement of brakes and the braking frequency are generally to be considered. According to optimal values of the force ratio  $p_B$  and the deceleration  $a_B$  the following criteria are usually taken into consideration when practically designing large-capacity and long belt conveyor systems:

- The required total braking force  $F_B$  must be calculated for the most unfavourable braking conditions which are determined by the filling ratio of the belt and by the distribution of the load in downhill and uphill segments of the conveyor. In this connection either the braking distance  $s_B$

or the braking time  $t_B$  must be specified. This will in turn determine the braking deceleration  $a_B$ .

- When taking values of the force  $F_B$  and the braking deceleration  $a_B$  as a basis for the design of the braking system and the belt conveyor, it has to be considered that the frictional contact between load and belt is maintained.
- It may be necessary to limit the total braking force to a given value  $F_{Bmax}$  in correspondence to a certain ratio  $p_{Bmax}$ , and thus to reduce the braking deceleration to a limiting value  $a_{Bmax}$ , in order to reduce the stresses on all parts of the installation, especially on the belt, and to maintain the friction grip on the braked pulleys.

The first criterion determines directly the braking deceleration  $a_B$ . Assuming a constant deceleration during the entire braking process, the following relationship between the braking time  $t_B$ , the braking distance  $s_B$  and the deceleration  $a_B$  ( $a_B < 0$ ) can be deduced:

$$s_B = \frac{1}{2} \cdot |a_B| \cdot t_B^2 = \frac{1}{2} \cdot v \cdot t_B \quad (15)$$

$$t_B = \sqrt{\frac{2 \cdot s_B}{|a_B|}} = 2 \cdot \frac{s_B}{v} \quad (16)$$

$$a_B = -2 \frac{s_B}{t_B^2} = -\frac{v}{t_B} \quad (17)$$

where :

$v$  = velocity

The last of these equations determines in connection with equations (4), (5), (6) and (7) the force ratio  $p_B$  and the peripheral forces  $F_B$  of the braked pulleys and thereby the belt stress during the braking process, if, as especially in the case of downhill belt conveyors ( $\sin \alpha < 0$ ), the deceleration  $a_B$  is not in contradiction with the second criterion. The latter defines the permissible acceleration  $a_B$  for fine-grained bulk materials in the following way (cf. standard DIN 22101):

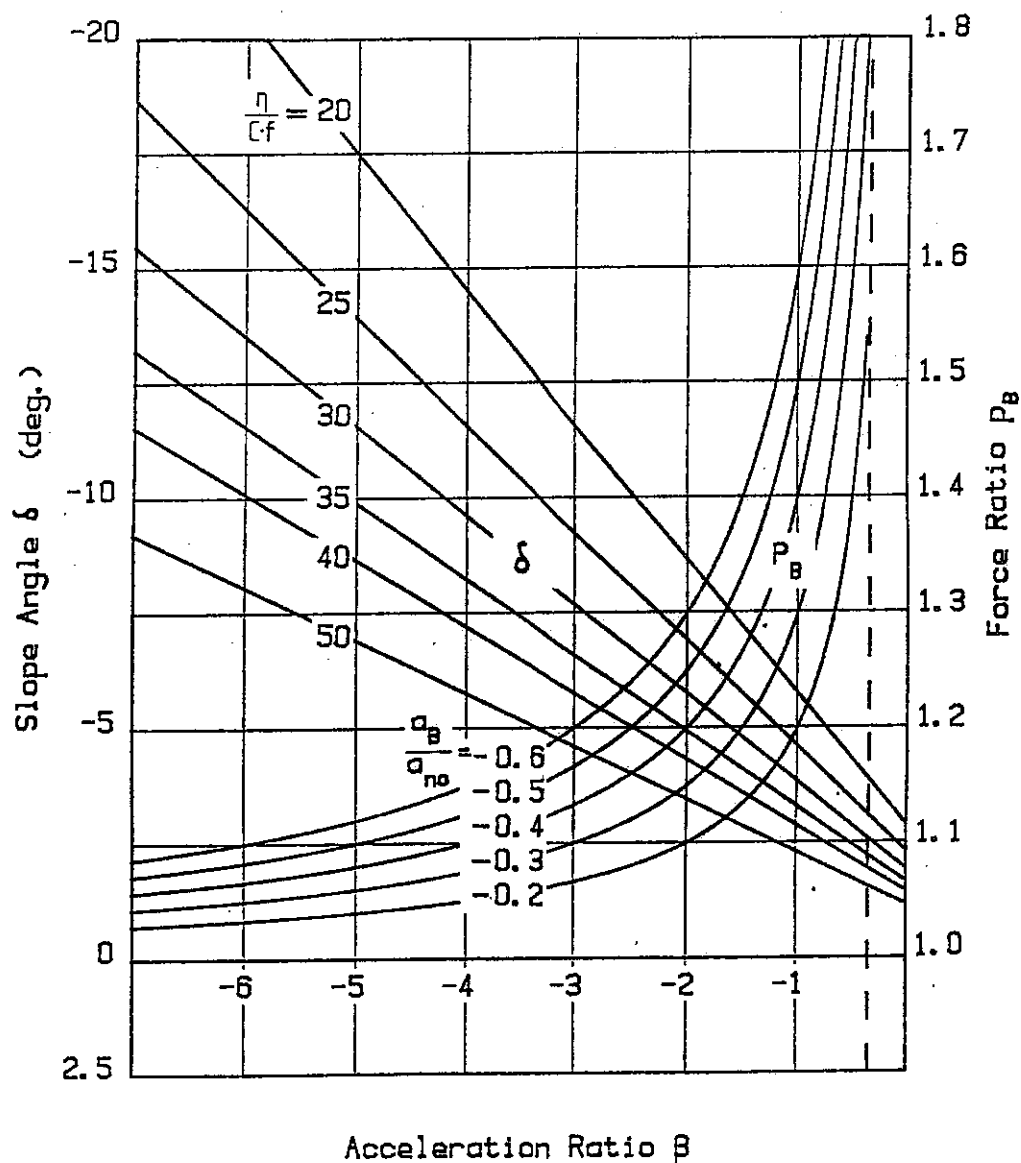


Figure 6 Diagram for the determination of the force ratio  $p_B$  as a function of the braking deceleration  $a_B$  and the acceleration ratio  $\beta$  for steep downhill belt conveyors  
(upper limiting value  $\beta_{max} = -0.35$  according to equation (7) for  $\sin \delta = -(2 - \eta) \cdot C \cdot f / \eta$  and  $\eta = 0.65$ )

$$|a_B| < |\mu \cdot \cos \delta + \sin \delta| \cdot g \quad (18)$$

$\mu$  = friction factor between load and belt

( $\delta < 0$  : downhill belt conveyors,  $\delta > 0$  : uphill belt conveyors)

In addition to this, possibly a compromise has to be found with regard to the values of the parameters  $s_B$ ,  $t_B$ , perhaps introduced too low, and the value of the force  $F_B$ , limited by the third criterion.

Figure 6 represents graphically the defining equation for the force ratio  $p_B$ :

$$p_B = 1 + \frac{a_B}{a_{no} \cdot \beta} \quad (19)$$

which demonstrates the influence of the acceleration ratio  $\beta$  and the relative deceleration  $a_B/a_{no}$ . Figure 6 is applicable to steep downhill belt conveyors with  $\sin \delta \leq -(2 - \eta) \cdot C \cdot f / \eta$  and thus  $\beta \leq 1 - (2 - \eta) = \eta - 1$  and shows, as well as figure 4 applicable to an optimization of the processes during the starting of belt conveyors, a hyperbolic dependence from the acceleration ratio  $\beta$ .

A great difference in the handling of figure 4 and figure 6 results from the fact that especially belt conveyors with higher operational belt velocities, according to safety and operational demands, are run with such values of parameters  $s_B$  or  $t_B$ , which correspond with force ratio  $p_B \geq 1.2$ , even in the case of higher conveyor slopes. All the other downhill belt conveyors have to be optimized with regard to the force ratio  $p_B$  and deceleration  $a_B$  analogously to the procedures described for the parameters  $p_A$  and  $a_A$  with respect to figure 5. In this connection, especially the type of the brake installation, its permissible thermal stress and its characteristics has to be considered.

#### 4. Conclusion

The dynamic behaviour of belt conveyor systems in their starting and braking phases is strongly determined by their drive and brake units. Especially in the case of large-capacity and long steeply inclined belt conveyors the optimum design of these units should take into account the values and the differences between the torque ratio  $p_{x0}$ , essentially determined by the characteristics of drive ( $x = A$ ) and brake units ( $x = B$ ), and the corresponding force ratio  $p_x$ , strongly determining the peripheral forces of the driven or braked pulleys and thus the belt stress as well as the acceleration  $a_A$  or deceleration  $a_B$  of belt conveyors. In the cases mentioned above the interaction between the parameters  $p_{x0}$ ,  $p_x$ ,  $a_x$  has to be carefully considered, if the corresponding figures and recommendations are taken out of handbooks or standards. In addition to this an optimum design of drive and brake units has to take into consideration the following aspects:

- the power requirement in the steady state of operation of the conveyor with the most unfavourable operational conditions,
- the force ratios  $p_A$  and  $p_B$  with respect to the mechanical stress of the belt and the starting and braking times of the conveyor,
- the starting and braking times of the conveyor with respect to the thermal stress of drive and brake units,

and in special cases (e.g. steeply inclined belt conveyors)

- the maximum acceleration and deceleration with respect to the frictional contact between belt and load.



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